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Proximal V_4-Magic Labeling

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Abstract

For a non-trivial Abelian group V_4 under multiplication a graph G is said to be V_4-magic graph if there exist a labeling g of the edges of G with non-zero elements of V_4 such that the vertex labeling g^* defined as $g^*(v) = \prod_u g(uv)$ taken over all edges uv incident at v is a constant.

If $g^*(v) = \prod_u g(uv)$ is constant for all vertices except for one or atmost two vertices $v \in V$, then the labeling is called Proximal V_4-magic labeling. The graph which admits Proximal V_4-magic labeling is calledas Proximal V_4-magic graph.

In this paper Proximal V_4-magic labeling for some special graphsand cycle related graphs are investigated.

Keyword: $U(m,n)$, $PF(m,n)$, Kite graph (n,t),$[P_m; C_n]$, $[P_m, C_n^{(t)}]$, DB_n, P_n, $P_n \odot K_1$, $P_n \odot mK_1$, $K_{1,n}$, $B_{n,n}$, $PT(n,m)$

AMS subject classification (2010): 05C78

1 Introduction

Laid Foundation by Euler in the 18th century. Graph Theory grew wider by Sedlack, Kong, Lee and Sun. Sedlack introduced Magic labeling, Bloom and Golomb connected graph labeling to a wide range of applicationsuch as coding theory, communication design, Radar, circuit design, Astronomy, Network and X-ray crystallography.

For a non-trivial Abelian group V_4 under multiplication a graph G is said to be V_4-magic graph if there exist a labeling g of the edges of G with non-zero elements of V_4 such that the vertex labeling $g*$ defined as $g*(v) = \prod_u g(uv)$ taken over all edges uv incident at v is a constant.

If $g*(v) = \prod_u g(uv)$ is constant for all vertices except for one or atmost two vertices $v \in V$, then the labeling is called Proximal V_4-magic labeling. The graph which admits Proximal V_4-magic labeling is calledas Proximal V_4-magic graph.

In this paper Proximal V_4-magic labeling for some special graphsand cycle related graphs are investigated.

2 Preliminaries

Definition 2.1 (Bistar)

$B_{n,n}$ is the graph obtained by joining the central (apex) vertex of twocopies of $K_{1,n}$ by an edge.

Definition 2.2 (Palm Tree)

A path of length t atached to the centre vertex of a star graph $K_{1,n}$ iscalled Palm Tree graph. It is denoted by $PT(n, t)$. It has $(n + t)$ edges.

Definition 2.3 (Umbrella)

The graph obtained by attaching one end vertex of a path P_m to the centre vertex of the Wheel or Cone W_n is called Umbrella graph and it denoted by $U(n, m), n \geq 3, m \geq 2$.

Definition 2.4 (Pedestal Fan Graph)

Let F_n be a fan and P_m be a path. The graph obtained by attaching the path to the center vertex u of the fan by an edge is called Pedestal Fan and denoted by $PF(n, m)$.

3 Main Results

Proximal V_4-magic labeling for Special Graphs

Definition 3.1 For a non-trivial Abelian group V_4 under multiplicationa graph G is said to be V_4-magic graph if there exist a labeling g of the edges of G with non-zero elements of V_4 such that the vertex labeling $g*$ defined as $g*(v) = \prod_u g(uv)$ taken over all edges uv incident at v is a constant.

If $g*(v) = \prod_u g(uv)$ is constant for all vertices except for one or atmost two vertices $v \in V$, then the labeling is called Proximal V_4-magiclabeling. The graph which admits Proximal V_4-magic labeling is called as Proximal V_4-magic graph.

Theorem 3.2

For $n \geq 3$, P_n becomes a Proximal V_4-magic graph

Proof.

Let G be a path graph P_n, $n \geq 3$.

Let $V(G) = \{v_p/1 \leq p \leq n\}$ be the vertex set of G

Let $E(G) = \{v_pv_{p+1}/1 \leq p \leq n-1\}$ be the edge set of G

Define a mapping $g : E(G) \rightarrow V_4 - \{1\}$ such that the edge labels are

$g(v_pv_{p+1}) = -1; 1 \leq p \leq n-1$

which induces $g* : V(G) \to V_4$ such that

$g*(v_1) = -1$ $g*(v_n) = -1$

$g*(v_p) = 1; 2 \leq p \leq n - 1$

Except the first vertex and last vertex, every other vertex get the constant

1. Hence P_n becomes a Proximal V_4-magic graph for $n \geq 3$

Illustration 3.3

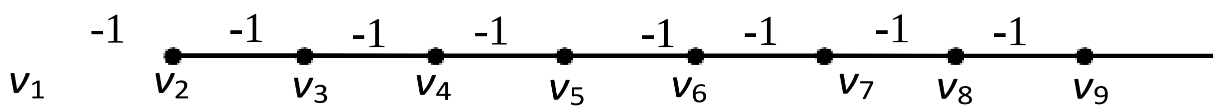

Figure 1: P_9

Theorem 3.4

$P_n \odot K_1$,the comb graph is Proximal V_4 for $n \geq 2$.

Proof.

Let G be a comb graph $P_n \odot K_1, n \geq 2$

Let $V(G) = \{v_p/1 \leq p \leq n\} \cup \{u_p/1 \leq p \leq n\}$ be the vertex set of G Let $E(G) =$

$\{v_pv_{p+1}/1 \leq p \leq n - 1\} \cup \{v_pu_p/1 \leq p \leq n\}$ be the edge setof G

Define a mapping $g : E(G) \to V_4 - \{1\}$ such that the edge labels are

$g(v_pv_{p+1}) = -1; 1 \leq p \leq n - 1$

$g(v_p u_p) = -1; 1 \le p \le n$

which induces $g* : V(G) \to V_4$ such that

$g*(v_1) = 1$

$g*(v_n) = 1$

$g*(v_p) = -1; 2 \le p \le n-1$

$g*(u_p) = -1; 1 \le p \le n$

All the vertex except v_1 an v_n get the constant -1.

Thus the comb graph satisfies the Proximal V_4-magic graph labeling andbecomes a Proximal V_4-magic graph.

Illustration 3.5

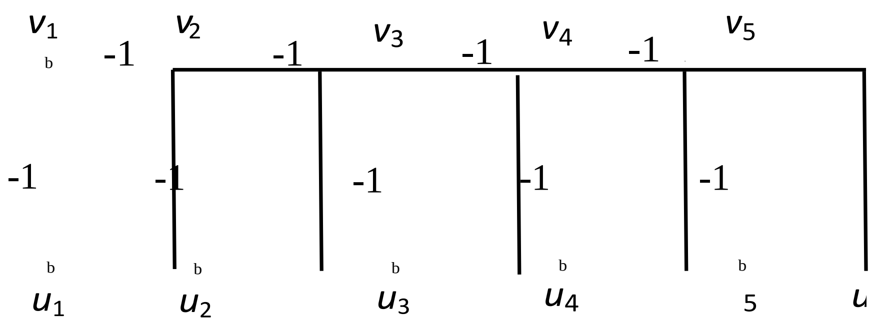

Figure 2: $P_5 \odot K_1$

Theorem 3.6

$P_n \odot mK_1$, the comb graph is Proximal V_4-magic graph for $m \in \mathbb{N}$.

Proof.

Let G be a $P_n \odot mK_1, m \in N$

Let $V(G) = \{v_p/1 \leq p \leq n\} \cup \{u_{pq}/1 \leq p \leq n, 1 \leq q \leq m\}$ be the vertexset of G

Let $E(G) = \{v_p v_{p+1}/1 \leq p \leq n\} \cup \{v_p u_{pq}/1 \leq p \leq n, 1 \leq q \leq m\}$ be theedge set of G

Define a mapping $g : E(G) \to V_4 - \{1\}$ such that the edge labels are

$g(v_p v_{p+1}) = -1; 1 \leq p \leq n - 1$

$g(v_p u_{pq}) = -1; 1 \leq p \leq n, 1 \leq q \leq m$ which induces $g* : V(G) \to V_4$ such thatwhen m is odd,

$g*(v_p) = 1$ for $p = 1, n$ $g*(v_p) = -1, 2 \leq p \leq n - 1$

$g*(u_{pq}) = -1; 1 \leq p \leq n, 1 \leq q \leq m$

when m is even,

$g*(v_p) = -1$ for $p = 1, n$ $g*(v_p) = 1, 2 \leq p \leq n - 1$

$g*(u_{pq}) = -1; 1 \leq p \leq n, 1 \leq q \leq m$

Thus in both cases except v_1 an v_n all other vertices get the same constantnumber either 1 or -1.

Thus G satisfies the Proximal V_4-magic graph labeling and $P_n \odot mK_1$ becomes a Proximal V_4-magic graph.

Illustration 3.7

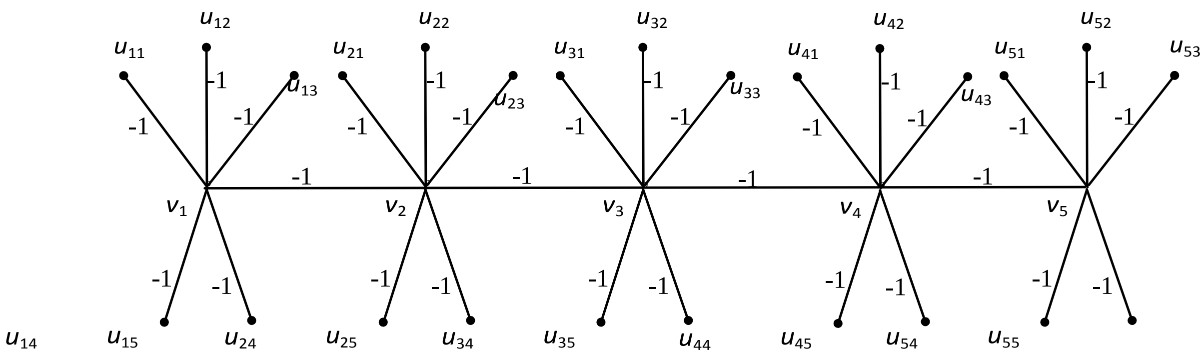

Figure 3: $P_5 \odot 5K_1$

Theorem 3.8

$K_{1,n}$ is Proximal V_4 if and only if $n \neq 4m + 1, m \in \mathbb{N}$.

Proof.

Let $n \neq 4m + 1, m \in \mathbb{N}$.

Let G be the star graph $K_{1,n}$

Let $V(G) = \{u, v_p/1 \leq p \leq n\}$ be the vertex set of GLet $E(G) = \{uv_p/1 \leq p \leq n\}$ be the edge set of G

Define a mapping $g : E(G) \rightarrow V_4 - \{1\}$ such that the edge labels are

$g(uv_p) = i; 1 \leq p \leq n$ and $n \neq 4m + 1, m \in \mathbb{N}$ which induces $g^* : V(G) \rightarrow V_4$ such that

$g^*(v_p) = i, 1 \leq p \leq n$ and $g^*(u)$ gets different values other than all othervertices.

Hence G becomes a Proximal V_4-magic graph by satisfying the Proximal V_4-magic graph labeling.

Conversely, let $K_{1,n}$ be a proximal V_4-magic graph.Let $n = 4m + 1, m \in \mathbb{N}$. By giving suitable edge labels to $K_{1,n}$, all the vertices $\{u, v_p/1 \leq p \leq n\}$ get the same constant.

$K_{1,n}$ becomes a V_4-magic graph, which is a contradiction to our assumption that $K_{1,n}$ is a proximal V_4-magic graph. Hence $n \neq 4m + 1, m \in \mathbb{N}$.

Thus $K_{1,n}$ is Proximal V_4 if and only if $n \neq 4m + 1, m \in \mathbb{N}$.

Illustration 3.9 $K_{1,5}$ (ie) $K_{1,4(1)+1}$ is not proximal V_4.

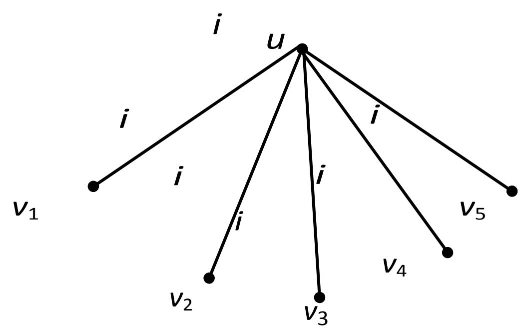

Figure 4:

$K_{1,10}$ is proximal V_4-magic graph.

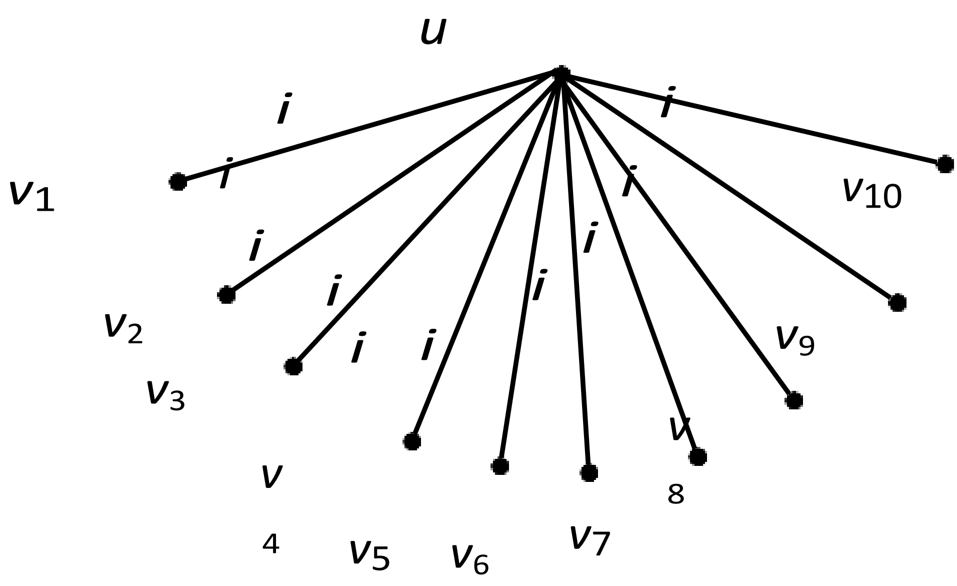

Figure 5: $K_{1,10}$

Theorem 3.10

Bistar graph $B_{n,n}$ is a Proximal V_4-magic graph for any n.

Proof.

Let G be a Bistar graph $B_{n,n}$, $n \in N$

Let $V(G) = \{u, u_p, v, v_p/1 \leq p \leq n\}$ be the vertex set of GLet $E(G) = \{uv, uu_p,$

$vv_p/1 \leq p \leq n\}$ be the edge set of G

Define a mapping $g : E(G) \to V_4 - \{1\}$ such that the edge labels are

$g(uv) = i$

$g(uu_p) = -1; 1 \leq p \leq ng(vv_p) = -1; 1 \leq p \leq n$

which induces $g* : V(G) \to V_4$ such that

all the vertices except u, v get the same vertex labeling.

Thus G satisfies the Proximal V_4-magic graph labeling. Hence Bistargraph $B_{n,n}$ is a Proximal V_4-magic graph for all n.

Illustration 3.11

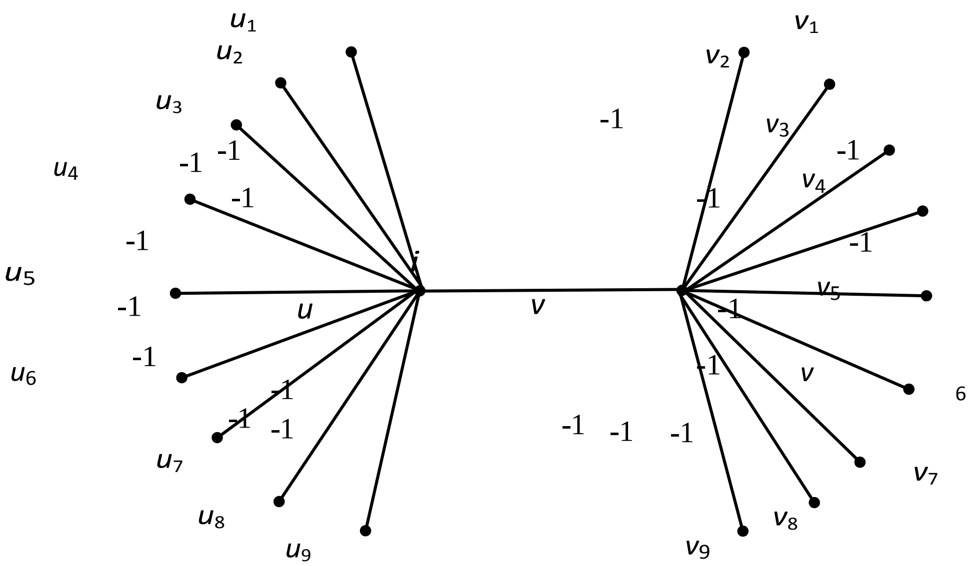

Figure 6: $B_{9,9}$

Theorem 3.12

Palm Tree $PT(n, m)$ is a Proximal V_4-magic graph for any $n, m \in \mathbb{N}$.

Proof.

Let G be $PT(n, m), n, m \in \mathbb{N}$

Let $V(G) = \{v_p, v, u_p / 1 \leq p \leq n, 1 \leq q \leq m\}$ be the vertex set of G

Let $E(G) = \{vv_p, vu_1, u_q u_{q+1} / 1 \leq p \leq n, 2 \leq q \leq m - 1\}$ be the edge setof G.

Define a mapping $g : E(G) \rightarrow V_4 - \{1\}$ such that the edge labels are

$g(vv_p) = -1; 1 \leq p \leq n g(vu_1) = i$

$g(u_q u_{q+1}) = i; 2 \leq q \leq m - 1$

which induces $g^* : V(G) \rightarrow V_4$ such that

when all the vertices except v and u_m get the same vertex labeling. Thus G satisfies the Proximal V_4-magic graph labeling. Hence Palm Tree$PT(n, m)$ is a Proximal V_4-magic graph for all $n, m \in \mathbb{N}$.

Illustration 3.13

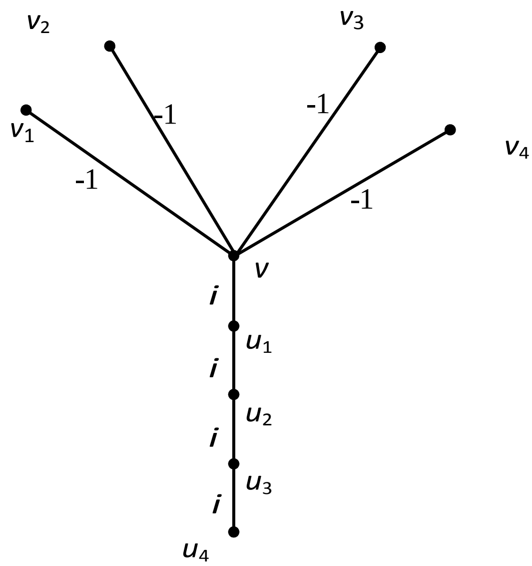

Figure 7: $PT(4, 4)$

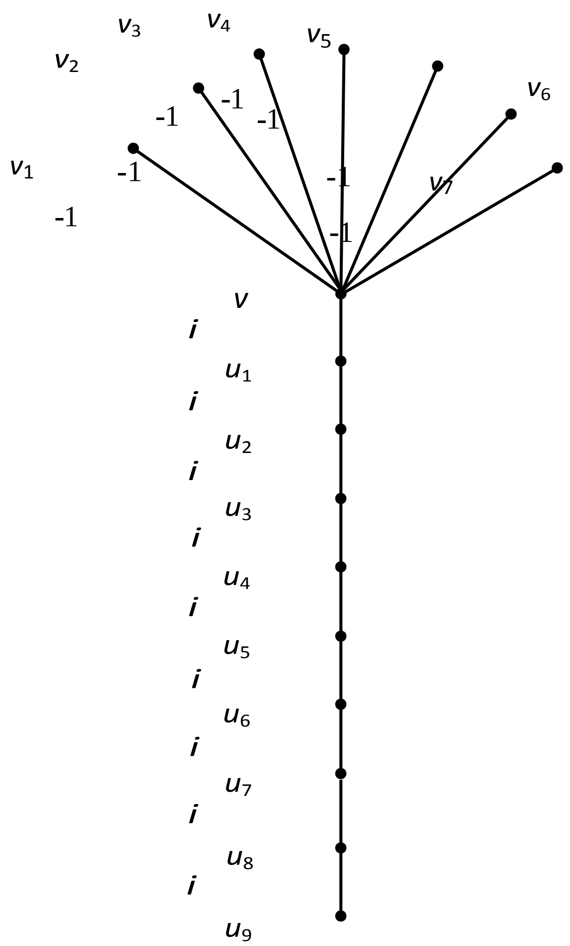

Figure 8: $PT\,(7, 9)$

Proximal V_4-Magic Labeling for Cycle Related Graphs

Theorem 3.14

Umbrella graph $U(m, n)$ is a Proximal V_4-magic graph.

Proof.

Let G be an Umbrella graph $U(m, n)$, for any $m, n \in \mathbb{N}$

Let $V(G) = \{u_p, v_q / 1 \leq p \leq m, 1 \leq q \leq n\}$ be the vertex set of G

Let $E(G) = \{u_p u_{p+1}, v_1 u_p / 1 \leq p \leq m - 1\} \cup \{v_q v_{q+1} / 1 \leq q \leq n - 1\}$ bethe edge set of G.

$$[u_{m+1} = u_1]$$

Define a mapping $g : E(G) \rightarrow V_4 - \{1\}$ such that the edge labels are

$g(v_p v_{p+1}) = -1; 1 \leq p \leq m - 1 g(u_p u_{p+1}) = i; 1 \leq p \leq m - 1$ $g(v_1 u_p) = -1; 1 \leq p \leq m$

$g(v_q v_{q+1}) = -1; 1 \leq q \leq n$

which induces $g^* : V(G) \rightarrow V_4$ such that the vertices v_1 and v_n get the different vertex labels, when m is even. And only v_n get different label when m is odd and all the other vertices gets the same vertex labels.

Hence $U(m, n)$ becomes a Proximal V_4-magic graph.

Illustration 3.15 $U(6, 5)$

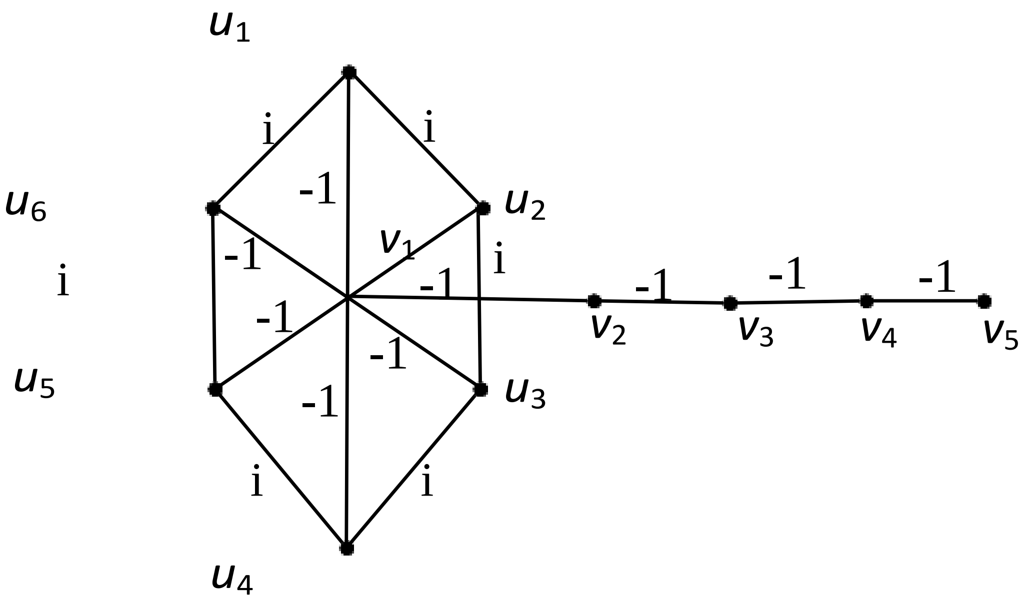

Figure 9: $U(6,5)$

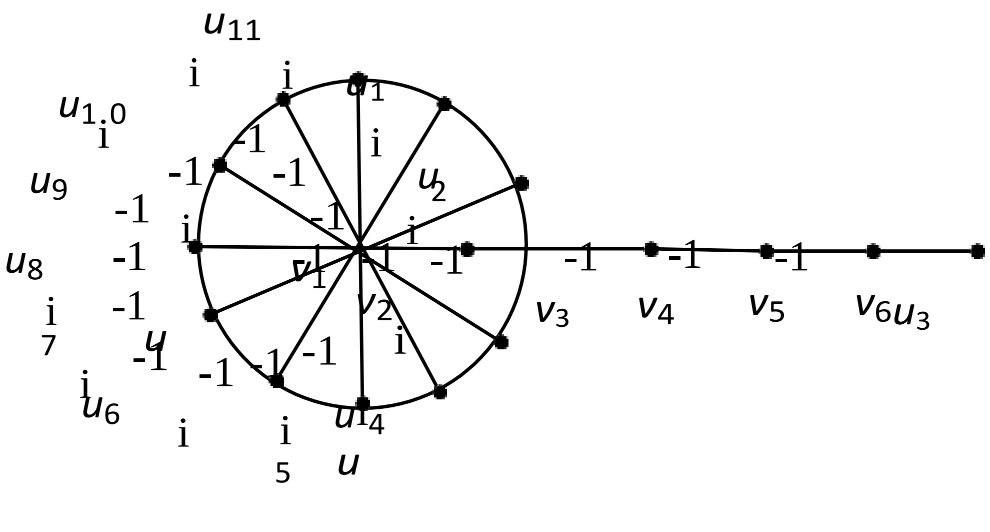

Figure 10: $U(11,6)$

Remark 3.16

Umbrella graph $U(m, n)$ becomes a Hefty V_4-magic graph, when m is odd.

Theorem 3.17

Pedestal Fan graph $PF(m, n)$ is a Proximal V_4-magic graph for $m \geq 3, n \geq 2$.

Proof.

Let G be Pedestal Fan graph $PF(m, n)$,$m \geq 3, n \geq 2$.

Let $V(G) = \{u, u_p, v_q/1 \leq p \leq m, 1 \leq q \leq n\}$ be the vertex set of G

Let $E(G) = \{uu_p/1 \leq p \leq m\} \cup \{u_pu_{p+1}/1 \leq p \leq m-1\} \cup \{uv_1\} \cup \{v_qv_{q+1}/1 \leq q \leq n\}$ be the edge set of G

Define a mapping $g : E(G) \rightarrow V_4 - \{1\}$ such that the edge labels are

$g(uu_1) = g(uu_m)) = -i$ $g(uu_p) = -1; 2 \leq p \leq m-1$ $g(u_pu_{p+1}) = i; 1 \leq p \leq m-1$

$g(v_qv_{q+1}) = -1; 1 \leq q \leq n-1 g(uv_1) = -1$

which induces $g* : V(G) \rightarrow V_4$ such that

When m is odd, only one vertex v_m get the different label and all othervertices have the same labels.

When m is even, two vertices u, v_m get different vertex label and all the other vertices get the constant vertex label.

Thus Pedestal Fan $PF(m, n)$ satisfies Proximal V_4-magic graph labelingand becomes a Proximal V_4-magic graph for $m \geq 3, n \geq 2$.

Illustration 3.18

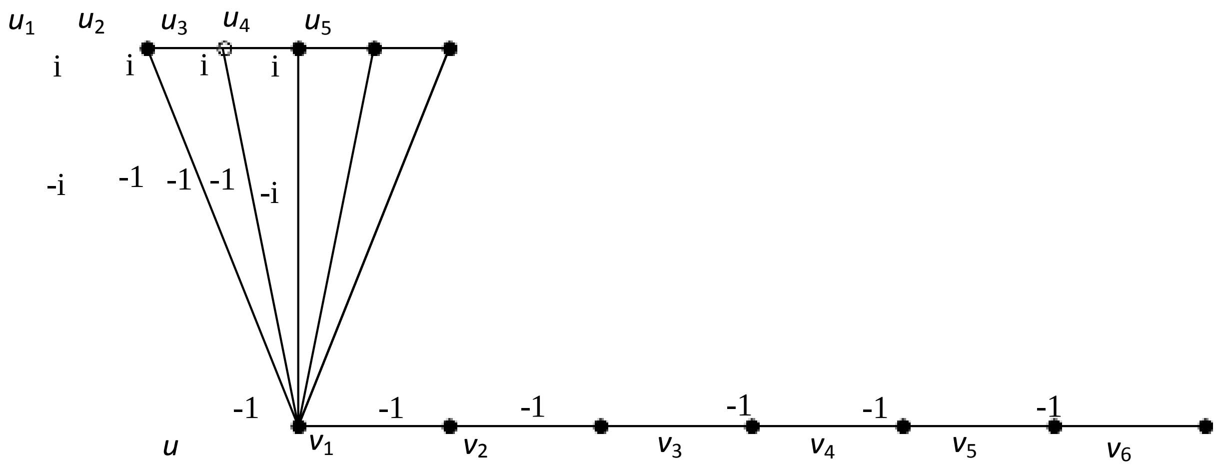

Figure 11: $PF(5, 6)$

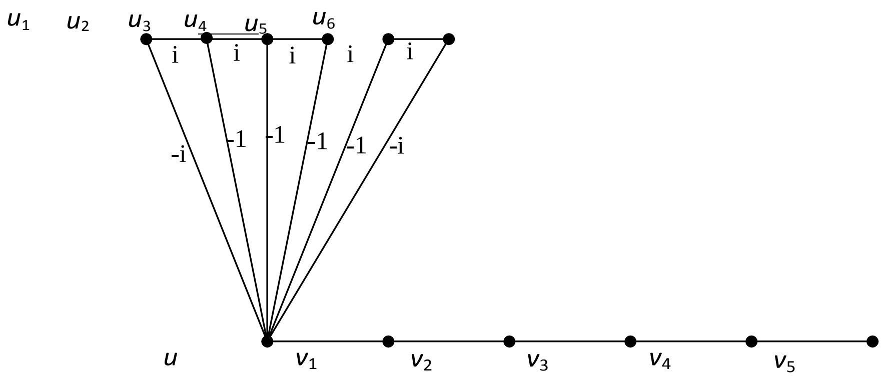

Figure 12: $PF(6, 5)$

Theorem 3.19

Kite graph (n, t) is a Proximal V_4-magic graph for $n \geq 3$ and $t \geq 1$.

Proof.

Let G be Kite graph (n, t),$n \geq 3$ and $t \geq 1$. Let $V(G) = \{v_p/1 \leq p \leq n\} \cup \{u_q/1 \leq q \leq t\}$ be the vertex set of G

Let $E(G) = \{v_pv_{p+1}/1 \leq p \leq n\} \cup \{v_1u_1\} \cup \{u_qu_{q+1}/1 \leq q \leq t-1\}$ bethe edge set of G

$[v_{n+1} = v_1]$

Define a mapping $g : E(G) \to V_4 - \{1\}$ such that the edge labels are

$g(vv_p) = -1; 1 \leq p \leq ng(v_1u_1) = -1$

$g(u_qu_{q+1}) = -1; 1 \leq q \leq t$

which induces $g* : V(G) \to V_4$ such that

except v_1 and u_t, all the other vertices get the same constant 1.

Thus Kite graph (n, t) becomes Proximal graph by satisfying the conditionof Proximal V_4-magic graph labeling.

Illustration 3.20 $(7, 5)$

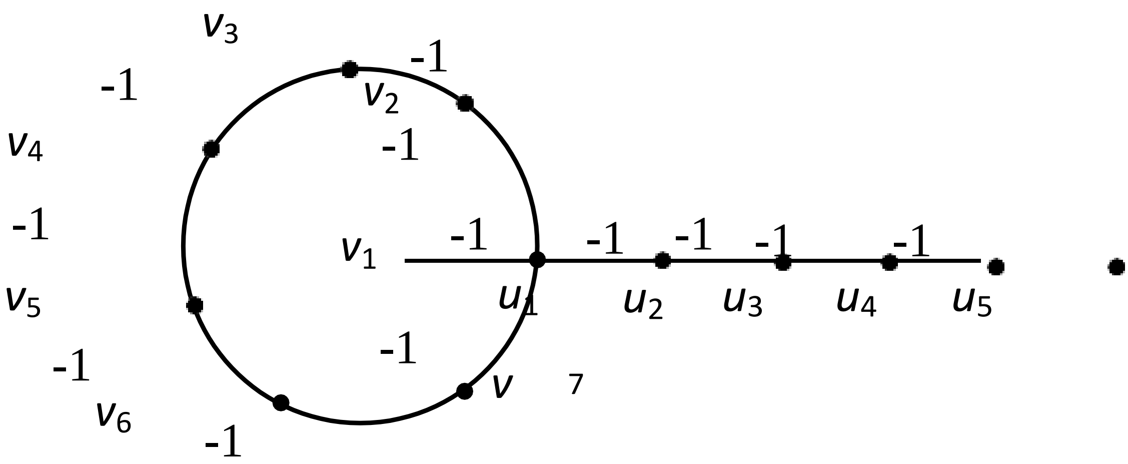

Figure 13:

Illustration 3.21 (10, 6)

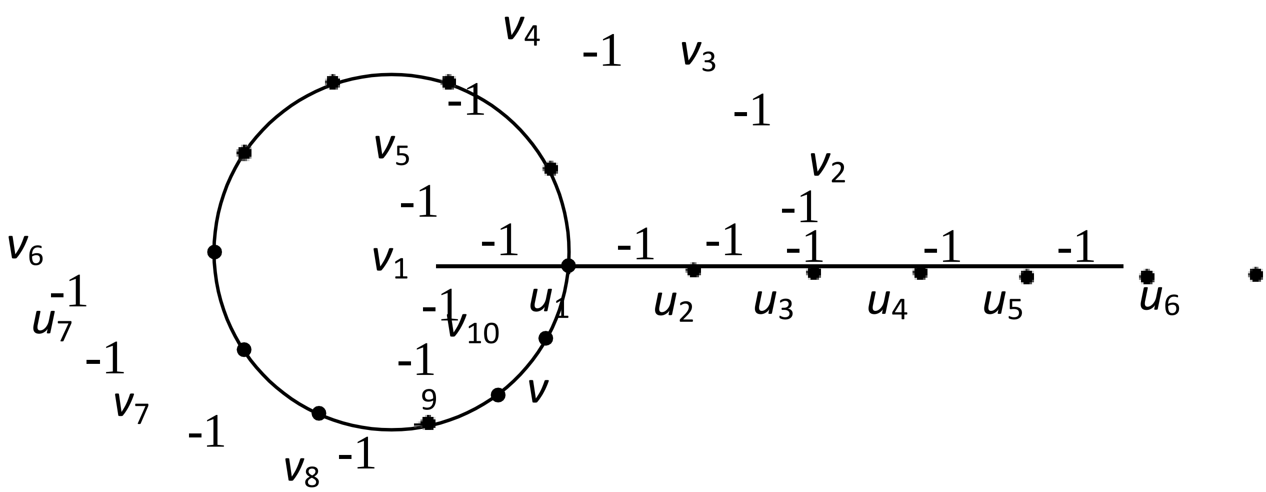

Figure 14:

Observation 3.22

Whether G is a Hefty V_4-magic graph or V_4-magic graph, $G \odot P_n$ is aProximal V_4-magic graph.

Theorem 3.23

$[P_m; C_n]$ graph is a Proximal V_4-magic graph for any $m \geq 2, n \geq 3$.

Proof.

Let G be a $[P_m; C_n]$ graph, $m \geq 2, n \geq 3$ Let $V(G) = \{v_{pq}/1 \leq p \leq m, 1 \leq q \leq n\}$ be the vertex set of G

Let $E(G) = \{v_{pq}v_{p+1,q}/1 \leq p \leq m-1, q = 1\} \cup \{v_{pq}v_{pq+1}/1 \leq p \leq m, 1 \leq q \leq n\}$ be the edge set of G.

$$[v_{pn+1} = v_{p1}]$$

Define a mapping $g : E(G) \to V_4 - \{1\}$ such that the edge labels are

$g(v_{pq}v_{p+1,q}) = -1; 1 \leq p \leq m-1$ and $q = 1$

$g(v_{pq}v_{pq+1}) = i, 1 \leq p \leq m$ and $1 \leq q \leq n$

which induces $g^* : V(G) \to V_4$ such that except v_{11} and v_{m1} all the othervertices get the same constant -1.

Thus $[P_m; C_n]$ graph becomes a Proximal V_4-magic graph by satisfyingthe condition of Proximal V_4-magic graph labeling.

Illustration 3.24 $[P_3; C_4]$

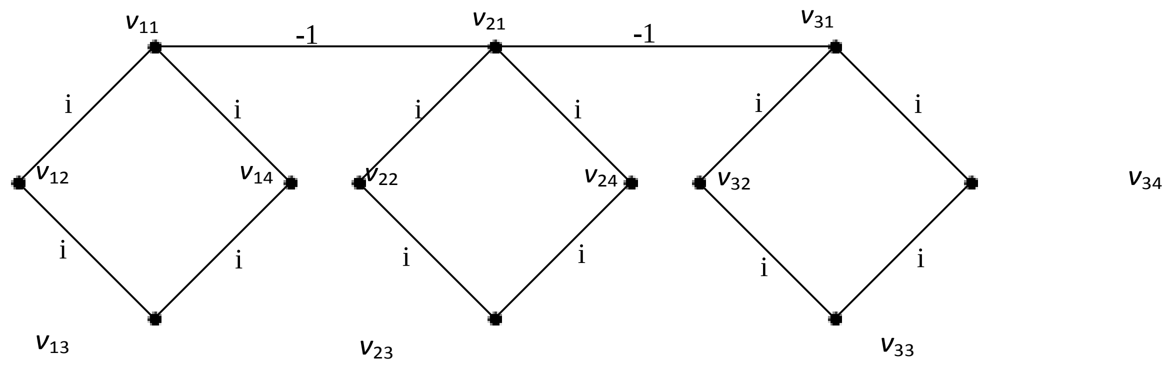

Figure 15: $[P_3; C_4]$

Illustration 3.25 $[P_4; C_5]$

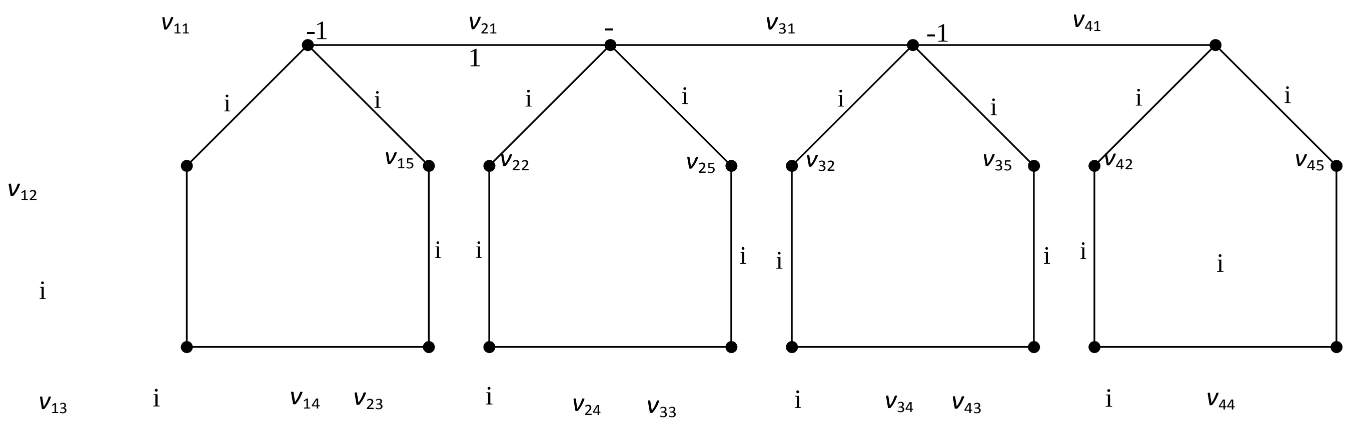

Figure 16: $[P_4; C_5]$

Theorem 3.26

$[P_n; C_m^{(t)}]$ is a Proximal V_4-magic graph for $n \geq 2, m \geq 3$ and $t \geq 1$.

Proof.

Let G be a $[P_n; C_m^{(t)}]$ graph, $n \geq 2, m \geq 3$ and $t \geq 1$.

Let n be odd or even

Case 1 Let both m and t be odd

Let $V(G) = \{u_p / 1 \leq p \leq n\} \cup \{v^{(t)}_{pq} / 1 \leq p \leq n, 1 \leq q \leq m-1$ and $t \geq 1\}$

be the vertex set of G

Let $E(G) = \{u_p u_{p+1} / 1 \leq p \leq n-1\} \cup \{v^{(t)}_{pq}, v^{(t)}_{pq+1} / 1 \leq p \leq n; 1 \leq q \leq m-2; t \geq 1\} \cup \{u_p v^{(t)}_{p1} / 1 \leq p \leq n, t \geq 1\} \cup \{u_p v^{(t)}_{p(m-1)} / 1 \leq p \leq n, t \geq 1\}$

be the edge set of G. Define a mapping $g : E(G) \to V_4 - \{1\}$ such thatthe edge labels are

$g(u_p u_{p+1}) = -1; 1 \leq p \leq n-1$

$g(u_p v^{(t)}_{p1}) = i, 1 \leq p \leq n, t \geq 1$

$g(u_p v^{(t)}_{p(m-1)}) = i, 1 \leq p \leq n, t \geq 1$

$g(v^{(t)}_{pq} v^{(t)}_{pq+1}) = -i, 1 \leq p \leq n, 1 \leq q \leq m-2,$ q is odd, $t \geq 1$

$g(v^{(t)}_{pq} v^{(t)}_{pq+1}) = -i, 1 \leq p \leq n, 1 \leq q \leq m-2,$ q is even, $t \geq 1$

which induces $g* : V(G) \to V_4$ such that

$g*(u_1) = 1$ and $g*(u_n) = 1$And all the other vertices

$g*(u_p) = -1, 2 \leq p \leq n-1$Hence G becomes a Proximal V_4-magic graph.

Case 2 Let both m and t be even.

Let $V(G) = \{u_p / 1 \leq p \leq n\} \cup \{v^{(t)}_{pq} / 1 \leq p \leq n, 1 \leq q \leq m-1$ and $t \geq 1\}$

be the vertex set of G

Let $E(G) = \{u_p u_{p+1} / 1 \leq p \leq n-1\} \cup \{v^{(t)}_{pq} v^{(t)}_{pq+1} / 1 \leq p \leq n; 1 \leq q \leq$

$m-2; t \geq 1\} \cup \{u_p v_{p1}^{(t)} / 1 \leq p \leq n, t \geq 1\} \cup \{u_p v_{p(m-1)}^{(t)} / 1 \leq p \leq n, t \geq 1\}$

be the edge set of G.

Define a mapping $g : E(G) \to V_4 - \{1\}$ such that the edge labels are

$g(u_p u_{p+1}) = -1; 1 \leq p \leq n-1$

$g(u_p v_{p1}^{(t)}) = i, 1 \leq p \leq n, t \geq 1$

$g(u_p v_{p(m-1)}^{(t)}) = -i, 1 \leq p \leq n, t \geq 1$

$g(v_{pq}^{(t)} v_{pq+1}^{(t)}) = -i, 1 \leq p \leq n, 1 \leq q \leq m-2,$ q is odd, $t \geq 1$

$g(v_{pq}^{(t)} v_{pq+1}^{(t)}) = i, 1 \leq p \leq n, 1 \leq q \leq m-2,$ q is even, $t \geq 1$

which induces $g^* : V(G) \to V_4$ such that

$g^*(u_1) = -1$ and $g^*(u_n) = -1$And all the other vertices

$g^*(u_p) = 1, 2 \leq p \leq n-1$Hence G becomes a Proximal V_4-magic graph.

Case 3 Let m be odd and t be even.

Let $V(G) = \{u_p / 1 \leq p \leq n\} \cup \{v_{pq}^{(t)} / 1 \leq p \leq n, 1 \leq q \leq m-1$ and $t \geq 1\}$

be the vertex set of G

Let $E(G) = \{u_p u_{p+1} / 1 \leq p \leq n-1\} \cup \{v_{pq}^{(t)} v_{pq+1}^{(t)} / 1 \leq p \leq n; 1 \leq q \leq m-2; t \geq 1\} \cup \{u_p v_{p1}^{(t)} / 1 \leq p \leq n, t \geq 1\} \cup \{u_p v_{p(m-1)}^{(t)} / 1 \leq p \leq n, t \geq 1\}$

be the edge set of G.

Define a mapping $g : E(G) \to V_4 - \{1\}$ such that the edge labels are

$g(u_p u_{p+1}) = -1; 1 \leq p \leq n-1$

$g(u_p v_{p1}^{(t)}) = i, 1 \leq p \leq n, t \geq 1$

$g(u_p v_{p(m-1)}^{(t)}) = i, 1 \leq p \leq n, t \geq 1$

$g(v_{pq}^{(t)} v_{pq+1}^{(t)}) = -i, 1 \leq p \leq n, 2 \leq q \leq m-2$, q is odd, $t \geq 1$

$g(v_{pq}^{(t)} v_{pq+1}^{(t)}) = i, 1 \leq p \leq n, 2 \leq q \leq m-2$, q is even, $t \geq 1$

which induces $g* : V(G) \rightarrow V_4$ such that

$g*(u_1) = -1$ and $g*(u_n) = -1$And all the other vertices

$g*(u_p) = 1, 2 \leq p \leq n-1$Hence G becomes a Proximal V_4-magic graph.

Case 4 Let m be even and t be odd

Let $V(G) = \{u_p/1 \leq p \leq n\} \cup \{v_{pq}^{(t)}/1 \leq p \leq n, 1 \leq q \leq m-1$ and $t \geq 1\}$

be the vertex set of G

Let $E(G) = \{u_p u_{p+1}/1 \leq p \leq n-1\} \cup \{v_{pq}^{(t)} v_{pq+1}^{(t)}/1 \leq p \leq n, 1 \leq q \leq m-2, t \geq 1\} \cup \{u_p v_{p1}^{(t)}/1 \leq p \leq n, t \geq 1\} \cup \{u_p v_{p(m-1)}^{(t)}/1 \leq p \leq n, t \geq 1\}$

be the edge set of G.

Define a mapping $g : E(G) \rightarrow V_4 - \{1\}$ such that the edge labels are

$g(u_p u_{p+1}) = -1; 1 \leq p \leq n-1$

$g(u_p v_{p1}^{(t)}) = i, 1 \leq p \leq n, t \geq 1$

$g(u_p v_{p(m-1)}^{(t)}) = -i, 1 \leq p \leq n, t \geq 1$

$g(v_{pq}^{(t)}v_{pq+1}^{(t)}) = -i, 1 \leq p \leq n, 2 \leq q \leq m-2,$ q is odd, $t \geq 1$

$g(v_{pq}^{(t)}v_{pq+1}^{(t)}) = i, 1 \leq p \leq n, 2 \leq q \leq m-2,$ q is even, $t \geq 1$

which induces $g* : V(G) \rightarrow V_4$ such that

$g*(u_1) = -1$ and $g*(u_n) = -1$And all the other vertices

$$g*(u_p) = 1, 2 \leq p \leq n-1$$

Hence G becomes a Proximal V_4-magic graph.

Thus from all the four cases $[P_n; C_m^{(t)}]$ is a Proximal V_4-magic graph is proved

Illustration 3.27

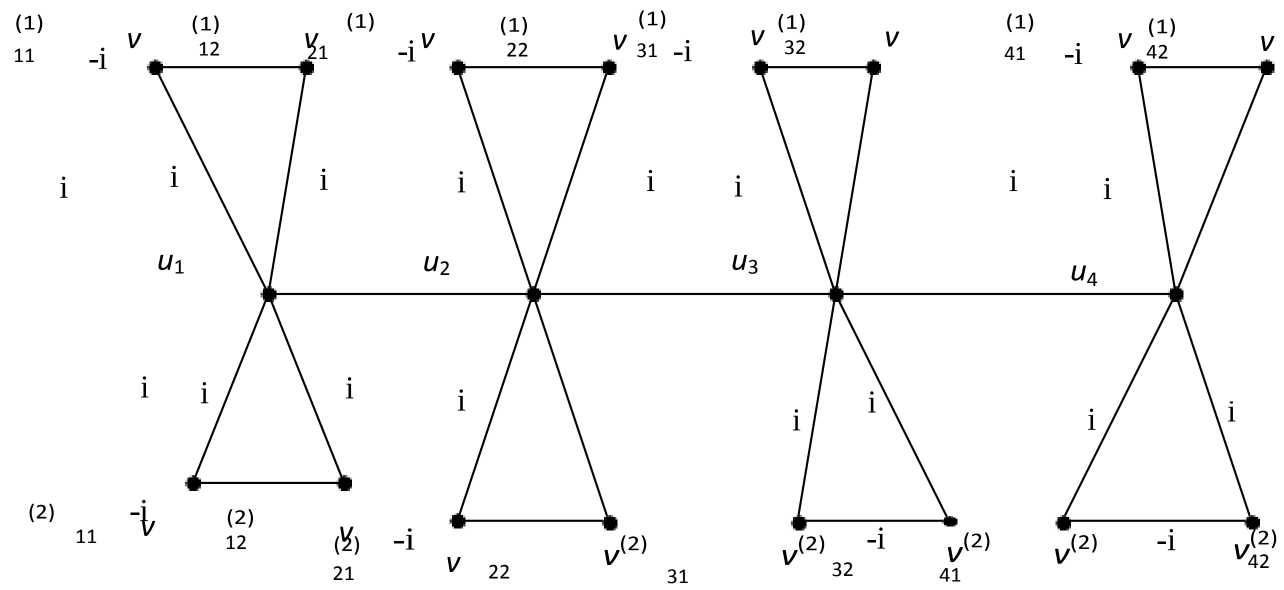

Figure 17: $[P_4; C^{(2)}]$

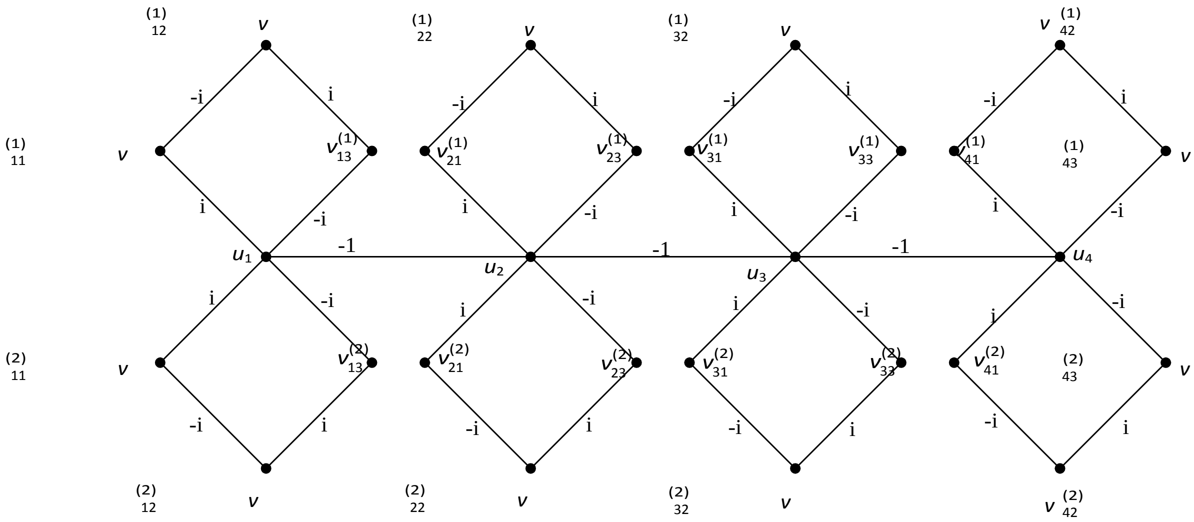

Figure 18: $[P_5; C_4^{(2)}]$

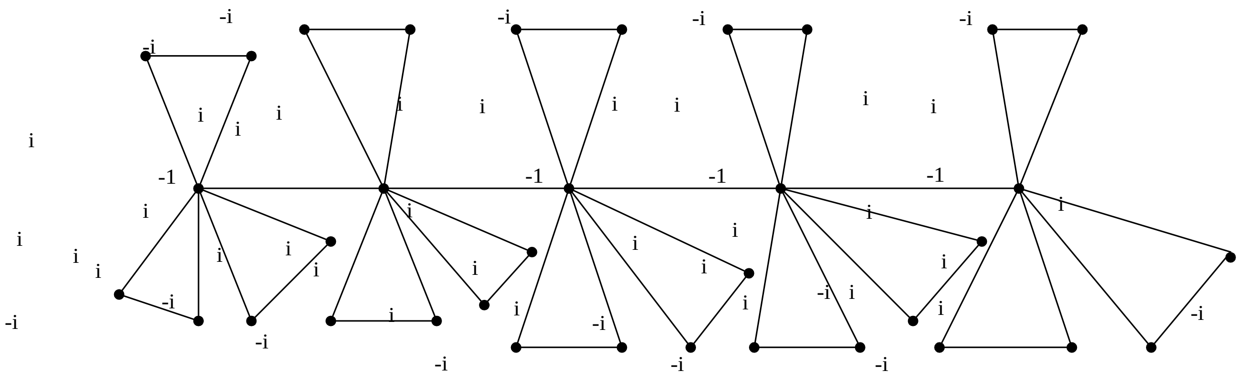

Figure 19: $[P_5; C_3^{(3)}]$

Remark 3.28

If all the edges of $[P_n; C_m^{(t)}]$ are labelled with -1, then again $[P_n; C_m^{(t)}]$ becomes a Proximal V_4-magic graph.

Theorem 3.29

Dumbell graph DB_n is a Proximal V_4-magic graph for $n \geq 3$.

Proof.

Let G be a Dumbell DB_n, $n \geq 3$

Let $V(G) = \{u_p/1 \leq p \leq n\} \cup \{v_p/1 \leq p \leq n\}$ be the vertex set of G Let $E(G) = \{u_p u_{p+1}/1 \leq p \leq n\} \cup \{v_p v_{p+1}, 1 \leq p \leq n\} \cup \{u_1 v_1\}$ be theedge set of G.

$$[u_{n+1} = u_1; v_{n+1} = v_1]$$

Define a mapping $g : E(G) \to V_4 - \{1\}$ such that the edge labels are

$g(u_p u_{p+1}) = i; 1 \leq p \leq n$ $g(v_p v_{p+1}) = i; 1 \leq p \leq n$ $g(u_1 v_1) = i$

which induces $g* : V(G) \to V_4$ such that

$g*(u_1) = -i, g*(v_1) = -i,$

$g*(u_p) = g*(v_p) = -1, 2 \leq p \leq n$

Hence DB_n becomes a Proximal V_4-magic graph as it satifies Proximal V_4-magic graph labeling.

Illustration 3.30

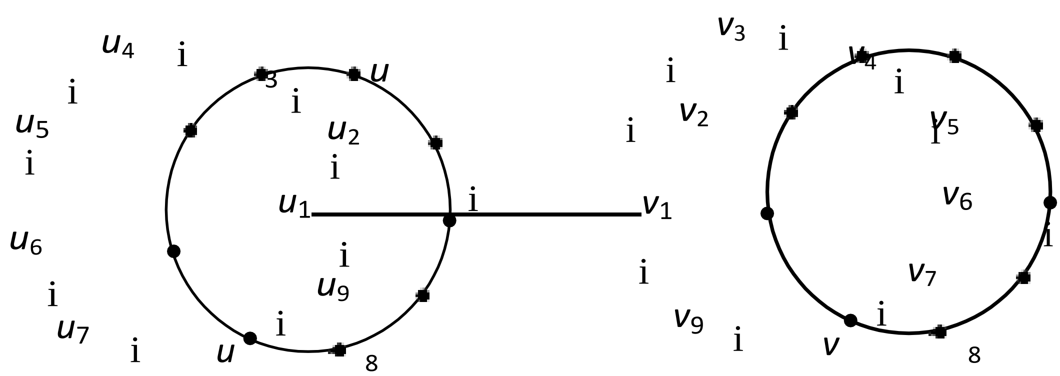

Figure 20:

Observation 3.31 G_1 and G_2 be any two Hefty V_4-magic graph by joining G_1 and G_2 by an edge the resulting graph becomes a Proximal V_4-magic graph.

Illustration 3.32

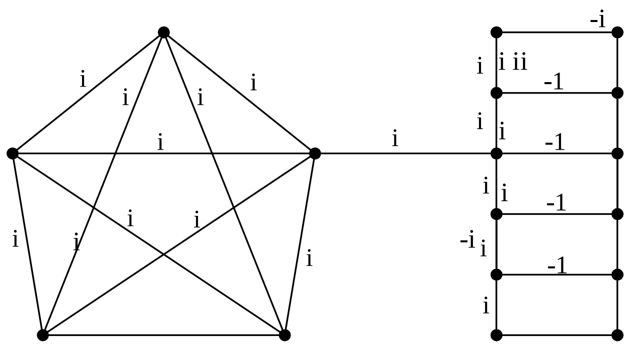

Figure 21:

References

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[3] J.A. Gallian, *A dynamic survey of graph labeling, Electronic Journal of Combinatorics 17*, D56,2010.